Spin Coulomb drag in the Hubbard chain

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The spin Coulomb drag is the decay of the spin current in a metal as a consequence of the Coulomb interaction between up- and down-spin carriers. This interaction reduces the spin current but does not change the spin-polarization. We calculate the critical exponents of the resistivity for up- and down-spin electrons and the transresistivity for the spin-polarized Hubbard chain with nonmagnetic impurities within the Kubo formalism using the Bethe ansatz solution and conformal invariance. Due to the Luttinger liquid properties the temperature dependence of the transport correlation functions follow power laws of T with nonuniversal exponents.

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I. INTRODUCTION

The spin Coulomb drag[1] is a distinctive feature of spin-polarized transport. Due to the friction between the spin-components via the Coulomb interaction the minority spins electrons are accelerated at the expense of the majority spin current.[2–6] In the absence of spin-flip mechanisms, such as magnetic impurities, the total magnetization is conserved and only the currents of up- and down-spin carriers changes. The spin Coulomb drag has been observed in a two-dimensional electron gas using optical methods:[7] an optical-helicity wave generates a wave of electron-spin polarization, whose time-evolution reveals the nature of the spin transport and relaxation. The spin-excitation relaxation time and the spin-diffusion constant are then determined from the analysis of the decay rate.[7] The spin Coulomb drag is a fundamental limitation for spintronics applications.

Many interesting effects are the result of Coulomb interactions between the carriers in low dimensions. In a one-dimensional (1D) conductor the correlations between electrons lead to exotic properties generically referred to as Luttinger liquid.[8] Characteristic of 1D metallic systems are the charge and spin separation, i.e., the charge and spin contents of the wave functions propagate with different velocities, and the disappearance of the Fermi liquid quasiparticle pole in the excitation spectrum, which is replaced by incoherent structures. Hence, the Fermi liquid picture breaks down for interacting electrons in 1D. Correlation functions usually acquire a power-law dependence on the frequency and temperature.

The spin Coulomb drag and related phenomena in a quantum wire have been studied by several authors.[9–14] In Ref. 9 the interplay of the spin-drag with the spin-charge separation is studied. The spin- and charge-density waves propagate at different speeds and relax with different damping rates. While the charges propagate ballistically, because the Coulomb interaction conserves momentum, the spin propagation is diffusive. In Ref. 10 quantum Monte Carlo simulations at finite temperature for the spin-drag conductance are conducted for the standard Hubbard chain with additional interactions, including magnetic impurities. Sonin[11] investigates the equilibrium spin currents at the edge states of the 2D Rashba medium, i.e., a spin-orbit quantum wire, and proposes a method to measure the spin-currents using a cantilever. In Ref. 12 the generation of a spin current by Coulomb drag between two quantum wires via the application of a magnetic field is studied using the bosonization method. The spin-charge separation in a strongly correlated spin-polarized chain is analyzed using path integrals and the bosonization technique in Ref. 13. A spin-polarized one-dimensional conductor could in principle be fabricated by etching a nanogroove into a locally depleted 2D electron gas in ferromagnetic Mn-doped GaAs.

In a previous calculation[14] we obtained the conductivity in the majority and minority bands and the transconductivity employing the bosonization approach and the Kubo formula. To obtain a finite transresistivity it is necessary to include impurity scattering and/or inelastic scattering by phonons. We did not include phonons and treated impurity scattering and the spin-flip Coulomb back-scattering perturbatively. A perturbative treatment in the disorder is insufficient to consider localization and dephasing effects, which are then completely neglected in the calculation.[14] The critical exponents of the power-law temperature dependence of the conductivities and the transconductivity are nonuniversal and strongly dependent on the forward-scattering amplitude.

In this paper we consider the current-current correlation functions derived in Ref. 14 and use conformal field theory and the Bethe *Ansatz* solution of the Hubbard model to obtain the critical exponents. The remainder of the paper is organized as follows. In Sec. II we briefly review the results of the Bethe *Ansatz* solution of the spin-polarized Hubbard chain and obtain the conformal towers.[15] In Sec. III summarize the generalized Kubo equation and the current-current correlation functions previously obtained in Ref. 14. The critical exponents of the resistivities are calculated in Sec. IV. A discussion of the results is presented in Sec. V and conclusions follow in Sec. VI.

II. BETHE ANSATZ SOLUTION OF THE HUBBARD MODEL

The model under consideration is the Hubbard chain consisting of nearest-neighbor tight-binding hopping and a local Coulomb repulsion between electrons of opposite spin,

$$H_U = -\sum_{i\sigma} (c^\dagger_{i\sigma} c_{i+1\sigma} + c^\dagger_{i+1\sigma} c_{i\sigma}) + U \sum_i n_{i\sigma} n_{i\bar{\sigma}}, \qquad (1)$$

where the hopping matrix element is set equal to one, and $n_{i\sigma}$ is the number operator. We consider N_e itinerant carriers on a chain of N_a sites with periodic boundary conditions.

Model (1) has been exactly diagonalized by means of two nested Bethe *Ansätze* by Lieb and Wu[16,17] in terms of two sets of rapidities, $\{k_j\}$ for $j=1,\cdots,N_e$ representing the charges, and $\{\lambda_\alpha\}$ for $\alpha=1,\ldots,M$, with N_e-2M corresponding to the population difference between the two spin components. These rapidities are self-consistently determined by the Bethe *Ansatz* equations and the energy is given by $E=-\Sigma_{j=1}^{N_e} 2\cos(k_j)$.

For $U>0$ all the rapidities are real in the ground state and densely distributed (without holes) between the respective Fermi points at $\pm Q$ for the charges and $\pm B$ for the "spinons."[16] In the absence of magnetization the "spinon" band is completely filled, so that $B=\infty$. In the thermodynamic limit, the densities for the rapidities, $\rho(k)$ and $\sigma(\lambda)$, satisfy the following integral equations[16,17]

$$\rho(k) = \frac{1}{2\pi} + \cos k \int_{-B}^{B} d\lambda a_1(\sin k - \lambda)\sigma(\lambda),$$

$$\sigma(\lambda) + \int_{-B}^{B} d\lambda' a_2(\lambda - \lambda')\sigma(\lambda') = \int_{-Q}^{Q} dk a_1(\lambda - \sin k)\rho(k), \qquad (2)$$

where the integration kernels are given by $a_n(x)=(Un/4\pi)/[x^2+(Un/4)^2]$. The integration limits are determined by the total number of carriers, $n=N_e/N_a=\int_{-Q}^{Q} dk\rho(k)$, and the magnetization, m, $2m=N_e/N_a-2M/N_a=N_e/N_a-2\int_{-B}^{B} d\lambda\sigma(\lambda)$. The total energy density is $E/N_a=-2\int_{-Q}^{Q} dk\cos(k)\rho(k)$.

The critical properties of correlation functions at low temperatures and small frequencies are determined by the low-energy excitation spectrum, which is given by the finite size corrections of mesoscopic order to the ground state energy in terms of a set of quantum numbers,[18,19]

$$E = N_a\epsilon_\infty + \sum_l \frac{\pi v_l}{2N_a}\left[\sum_q (\hat{z}^{-1})_{lq}\Delta N_q\right]^2 + \sum_l \frac{2\pi v_l}{N_a}\left\{\left[\sum_q z_{ql} D_q\right]^2 + n_l^+ + n_l^- - \frac{1}{12}\right\}, \qquad (3)$$

where ϵ_∞ is the ground state energy density in the thermodynamic limit, l and q label the two rapidity bands and take values c and s (for charges and "spinons"), and v_l denote the group velocities of the two rapidity bands. Here ΔN_q is the departure of the number of rapidities in the band q from the equilibrium value, i.e., ΔN_e and ΔM, respectively. Note that each band has two Fermi points corresponding to forward and backward moving states. D_q is the backward-scattering quantum number, i.e., $2D_q$ represents the difference of forward to backward moving states in each band. These quantities are sensitive to the parity in each set of rapidities,[20]

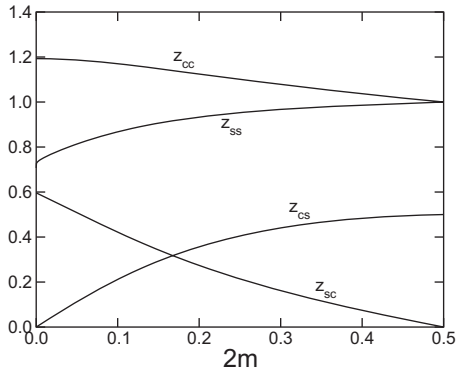

FIG. 1. Dressed generalized charges for $U=4$ and $n=0.5$ as a function of the magnetization m. Note the singular behavior of z_{ss} as $m\to 0$, where $z_{ss}=1/\sqrt{2}$ (Ref. 15).

$$D_c = (\Delta N_e + \Delta M)/2 \quad (mod\ 1),$$

$$D_s = \Delta N_e/2 \quad (mod\ 1). \qquad (4)$$

Finally, $n_q^\pm$ define the low-lying particle-hole excitations about each of the Fermi points. Here ΔN_q, $n_q^\pm$ and $2D_q$ take always integer values; hence D_q can either be an integer or half-integer depending on the ΔN_q.

The quantities z_{lq} in Eq. (3) are the dressed generalized charges of the excitations, which describe the interplay of the different Fermi points when particles (charges or "spinons") are added or removed, and $\hat{z}^{-1}$ denotes the inverse of the matrix. The group velocities and the dressed generalized charges are obtained from the Bethe *Ansatz*. Here we only need the charges $z_{lc}=\xi_{l,c}(Q)$ and $z_{ls}=\xi_{l,s}(B)$, but not the group velocities v_l. The functions $\xi_{l,q}$ satisfy coupled integral equations similar to Eq. (2), namely,[18]

$$\xi_{l,c}(k) = \delta_{l,c} + \int_{-B}^{B} d\lambda a_1(\sin k - \lambda)\xi_{l,s}(\lambda),$$

$$\xi_{l,s}(\lambda) = \delta_{l,s} - \int_{-B}^{B} d\lambda' a_2(\lambda - \lambda')\xi_{l,s}(\lambda') + \int_{-Q}^{Q} dk\cos k a_1(\lambda - \sin k)\xi_{l,c}(k). \qquad (5)$$

The equations simplify considerably for $B=\infty$ (zero magnetization): $z_{cs}=0$, $z_{ss}=1/\sqrt{2}$, $z_{sc}=\frac{1}{2}z_{cc}$, while z_{cc} is determined through a single integral equation.[18,19] The quantity $\theta=2z_{cc}^2$ is related to the charge stiffness. In this limit, the Fermi momentum for the charges is $p_{Fc}=\pi n$ with $n=N_e/N_a$ and the one for the spinons $p_{Fs}=\pi n/2$, so that for the Fermi momentum of up- and down-spin electrons we have $p_{F1}=p_{F2}=\pi n/2$.

In general the coupled Eqs. (5) have to be solved numerically. The dependence of z_{lq} as a function of $2m=N_e/N_a-2M/N_a$ is shown for $U=4$ and $n=N_e/N_a=0.5$ in Fig. 1. The dependence for $U=2$ and $U=8$ is similar and is not shown here. z_{cc} for $2m\to 0$ is nonuniversal and varies between $\sqrt{2}$ for $U=0$ and 1 for $U\to\infty$. Note that z_{ss} is singular as m tends to zero. This singularity arises from a $1/\ln(m)$-dependence,

consequence of the SU(2) invariance, which has been discussed in detail by solving the corresponding Wiener-Hopf equation in Ref. 15. The other three generalized charges are affected by this singularity only in higher derivatives. For $n=0.5$ and $2m\to 1/2$, on the other hand, only z_{cs} is nonuniversal (U-dependent).

In terms of the quantum numbers defined above, the total momentum of the system is given by[18,19]

$$P=\frac{2\pi}{N_a}\sum_l [N_l D_l + n_l^+ - n_l^-]. \tag{6}$$

The conformal dimensions of primary fields characterized by the above quantum numbers are obtained from Eqs. (3) and (6)[18,20]

$$2\Delta_c^{\pm}=2n_c^{\pm}+\left[z_{cc}D_c+z_{sc}D_s\pm\frac{z_{ss}\Delta N_e-z_{cs}\Delta M}{2\ det}\right]^2,$$

$$2\Delta_s^{\pm}=2n_S^{\pm}+\left[z_{cs}D_c+z_{ss}D_s\pm\frac{z_{cc}\Delta M-z_{sc}\Delta N_e}{2\ det}\right]^2, \tag{7}$$

where $det=z_{cc}z_{ss}-z_{sc}z_{cs}$.

To calculate the correlation function $\langle \mathcal{O}(x,t)\mathcal{O}^\dagger(0,0)\rangle$ first the allowed sets of quantum numbers associated with the operator $\mathcal{O}$ and the conformal dimensions have to be determined. Each set of quantum numbers gives rise to one term for the correlation function. At finite temperature and for finite chain length each term of the correlation function for long times and large distances is proportional to[17,19,20]

$$\begin{aligned}&\exp[-2i(D_c p_{Fc}+D_s p_{Fs})]\\&\times\left\{\frac{\pi T/N_a}{\sinh[\pi T(x-iv_c t)/v_c]}\right\}^{2\Delta_c^+}\\&\times\left\{\frac{\pi T/N_a}{\sinh[\pi T(x+iv_c t)/v_c]}\right\}^{2\Delta_c^-}\\&\times\left\{\frac{\pi T/N_a}{\sinh[\pi T(x-iv_s t)/v_s]}\right\}^{2\Delta_s^+}\\&\times\left\{\frac{\pi T/N_a}{\sinh[\pi T(x+iv_s t)/v_s]}\right\}^{2\Delta_s^-},\end{aligned} \tag{8}$$

where the exponential in the first line represents the momentum transfer across the Fermi surfaces. Here the time t is the Euclidean time. As expected for a Luttinger liquid, at $T=0$ the response function falls off with a power law for long times and distances, while at finite T this dependence is exponential.

III. CURRENT-CURRENT CORRELATION FUNCTION

In this section we briefly summarize the results of Ref. 14 we need for the present calculation. The current operator for up- and down-spin electrons is defined as

$$j_s=\sum_k \frac{k}{m^*}(a_{ks}^\dagger a_{ks}+b_{ks}^\dagger b_{ks}), \tag{9}$$

where a_{ks} ($k>0$) and b_{ks} ($k<0$) are the annihilation operators for forward and backward moving electrons, respectively. We introduced an effective mass m^* (assumed to be the same for up- and down-spin carriers) and linearized in the momentum. In terms of the spin-components the conductivity is defined as a 2×2 matrix

$$\sigma_{s,s'}(z)=-i(e^2/z)\chi_{s,s'}(z)+i(e^2 n_s/m^* z)\delta_{s,s'}, \tag{10}$$

where e is the electron charge, n_s the number of carriers with spin-component s and $\chi_{s,s'}(z)=-\langle\langle j_s;j_{s'}\rangle\rangle_z$ is the current-current correlation function.[1,21] The diagonal terms of $\hat{\sigma}$ are the conductivity of the up- and down-spins, respectively, and the off-diagonal component is the transconductivity. It is then possible to define relaxation rates[21]

$$\frac{1}{\tau_{s,s'}}=-\lim_{\omega\to 0}\omega\langle\langle j_s;j_{s'}\rangle\rangle''_\omega\frac{m^*}{n_{s'}}, \tag{11}$$

where $\tau_{s,s}$ refers to the relaxation times of the conductivity and $\tau_{s,\bar{s}}$ to that of the transconductivity. Here $\langle\langle;\rangle\rangle''_\omega$ refers to the imaginary part of the correlation function.

The scattering of the electrons off impurities is defined by the Hamiltonian

$$H_{imp}=\frac{\lambda}{L}\sum_{jkk's} e^{i(k'-k)R_j}[a_{ks}^\dagger a_{k's}+b_{ks}^\dagger b_{k's}+a_{ks}^\dagger b_{k's}+b_{ks}^\dagger a_{k's}], \tag{12}$$

where λ is the coupling strength, which is assumed to be weak, and the R_j denote the positions of the scattering centers. It is assumed that the impurities are distributed at random and that their concentration is low. We disregard the interference among the scattering centers.[21] This neglects any possibility for a localization of states due to disorder.

It should be pointed out that there exists another class of (magnetic) impurities which can be incorporated into the Hubbard chain without destroying the integrability.[22] There are two variants; they can be embedded into the chain or placed at one end-point of the chain.[23] Only the embedded ones are meaningful for the conductivity. These impurities are very different from the nonmagnetic impurities considered in Eq. (12), in particular, because the scattering conserves the momenta of the particles and hence does not produce a momentum transfer of $2k_F$. A finite concentration of the integrable impurities have been shown to modify the dressed generalized charges z_{lq} and hence the critical exponents of correlation functions.[24,25]

The force operator acting on the current of spin component s is

$$\begin{aligned}A_s&=[j_s,H_{imp}]\\&=\frac{\lambda}{m^* L}\sum_{jkk'}(k-k')e^{i(k'-k)R_j}\\&\quad\times(a_{ks}^\dagger a_{k's}+b_{ks}^\dagger b_{k's}+a_{ks}^\dagger b_{k's}+b_{ks}^\dagger a_{k's}).\end{aligned} \tag{13}$$

Equation (13) has two types of terms; the first two terms involve only forward scattering, i.e., small momentum transfer, which can be neglected, while the latter two terms consist of backward scattering, i.e., across the Fermi surface with momentum transfer $2k_{Fs}$.[26] To second order in λ the imaginary part of the equal spin current-current correlation function is then[14]

$$\omega^2\langle\langle j_s;j_s\rangle\rangle''_\omega = -\left(\frac{\lambda}{m^*L}\right)^2 n_i \sum_{kk'} (k-k')^2 \times [\langle\langle a^\dagger_{ks} b_{k's}; b^\dagger_{k's} a_{ks}\rangle\rangle''_\omega + \langle\langle b^\dagger_{ks} a_{k's}; a^\dagger_{k's} b_{ks}\rangle\rangle''_\omega], \tag{14}$$

where n_i/L is the impurity density. The momentum transfer for these terms is of $2k_{Fs}$. Evaluated for the noninteracting system ($U=0$) these terms yield $1/\tau_{s,s}=2\lambda^2 n_i m^*/\pi n_s$.[14,21,26]

The transconductivity is given by the spin-up current spin-down current correlation function[14]

$$\begin{aligned}\omega^2\langle\langle j_s;j_{\bar s}\rangle\rangle''_\omega = &-\frac{\lambda U}{(m^*L)^2}\sum_{jkk'q}(k-k')^2 \\ &\times [e^{i(k'-k)R_j}\langle a^\dagger_{k+q\bar s} b_{k'+q\bar s}\rangle\langle\langle a^\dagger_{ks} b_{k's}; b^\dagger_{k's} a_{ks}\rangle\rangle''_\omega \\ &+ e^{-i(k'-k)R_j}\langle b^\dagger_{k-q\bar s} a_{k'-q\bar s}\rangle\langle\langle b^\dagger_{ks} a_{k's}; a^\dagger_{k's} b_{ks}\rangle\rangle''_\omega \\ &+ e^{-i(k'-k)R_j}\langle b^\dagger_{k'-qs} a_{k-qs}\rangle\langle\langle a^\dagger_{k\bar s} b_{k'\bar s}; b^\dagger_{k'\bar s} a_{k\bar s}\rangle\rangle''_\omega \\ &+ e^{i(k'-k)R_j}\langle a^\dagger_{k'+qs} b_{k+qs}\rangle\langle\langle b^\dagger_{k\bar s} a_{k'\bar s}; a^\dagger_{k'\bar s} b_{k\bar s}\rangle\rangle''_\omega],\end{aligned} \tag{15}$$

where we factorized in the usual way

$$a^\dagger_{k+qs} b^\dagger_{k'-q\bar s'} a_{k\bar s'} b_{k's'} \to \langle a^\dagger_{k+qs} b_{k's'}\rangle b^\dagger_{k'-q\bar s'} a_{k\bar s'} + \langle b^\dagger_{k'-q\bar s'} a_{k\bar s'}\rangle a^\dagger_{k+qs} b_{k's'}. \tag{16}$$

Each of the factors conserves the spin and the expectation values are evaluated using H_{imp}. The sum over j just yields the number of impurities, n_i, where again we neglect the interference in the scattering between impurities. In all terms the momentum transfer in the scattering is across the Fermi surface, i.e., either $\pm 2k_{F\uparrow}$ or $\pm 2k_{F\downarrow}$.

When the correlation functions and expectation values on the right-hand side of Eq. (15) are evaluated for the noninteracting system we obtain

$$\omega\langle\langle j_s;j_{\bar s}\rangle\rangle''_\omega = -\frac{\lambda^2 U n_i m^*}{\pi^2}\left(\frac{1}{k_{F\bar s}}+\frac{1}{k_{Fs}}\right), \tag{17}$$

i.e., the transconductivity is proportional to the impurity concentration, to the Coulomb interaction, the square of the impurity scattering and inversely proportional to the group velocities.

IV. CRITICAL EXPONENTS

In this section we calculate the critical exponents for the resistivities and the transresistivity using the Bethe *Ansatz* solution of the Hubbard model and conformal invariance.

We consider first the conductivity of the majority spin (up-spin) carriers. According to Eq. (14) the relevant operator for the relaxation is $a^\dagger_{k\uparrow} b_{k'\uparrow}$ and its Hermitian conjugated. This corresponds to momentum transfer from one Fermi point to the other, i.e., $\pm 2k_{F\uparrow}$. The corresponding quantum numbers for this process are $\Delta N_e=\Delta M=0$ and $D_c=-D_s=\pm 1$. The momentum transfer is then $\Delta P=2D_c k_{F\uparrow}+2(D_c+D_s)k_{F\downarrow}=\pm 2k_{F\uparrow}$. The leading critical exponents correspond to no excitations at the Fermi points, i.e., $n_c^\pm=n_s^\pm=0$, so that

$$\theta_\uparrow = 4(\Delta_c+\Delta_s) = 2(z_{cc}-z_{sc})^2 + 2(z_{cs}-z_{ss})^2. \tag{18}$$

Similarly, the relevant operators for the current-current correlation function of down-spin (minority spin) electrons are $a^\dagger_{k\downarrow} b_{k'\downarrow}$ and $b^\dagger_{k\downarrow} a_{k'\downarrow}$. This corresponds to a momentum transfer of $\pm 2k_{F\downarrow}$. The quantum numbers for these operators are $\Delta N_e=\Delta M=0$, $D_c=0$, and $D_s=\pm 1$. For the leading critical exponents there are no excitations at the Fermi points, $n_c^\pm=n_s^\pm=0$, and

$$\theta_\downarrow = 4(\Delta_c+\Delta_s) = 2z_{sc}^2 + 2z_{ss}^2. \tag{19}$$

Instead of the imaginary Euclidean time we consider real time ($it\to t$) and the limit $|x|\ll v|t|$, i.e., the long-time approximation (we need the $\omega\to 0$ limit) and relatively short distances. The correlation functions are then proportional to (D is an electronic cutoff for the long-time approximation of the order of the hopping integral)

$$-\frac{iD^2}{\pi}\int dt\,\sin(\omega t)\left[\frac{-i\pi T/D}{\sinh(\pi T t)}\right]^{\theta_s} = \frac{D^2}{2\pi^2 T}\frac{1}{\Gamma(\theta_s)}\left(\frac{2\pi T}{D}\right)^{\theta_s}\sinh(\omega/2T)\left|\Gamma\left(\frac{\theta_s}{2}+i\frac{\omega}{2\pi T}\right)\right|^2. \tag{20}$$

Here Γ is the Gamma function. For the noninteracting system and zero polarization $\theta_s=2$. Hence, we rewrite the exponent as $\theta_s=2+\eta_s$ and expand Eq. (20) in powers of η_s

$$\begin{aligned}&2T\left(\frac{2\pi T}{D}\right)^{\eta_s}\sinh(\omega/2T)\exp[-\ln\Gamma(2+\eta_s)] \\ &\quad\times\exp\left[2\,\mathrm{Re}\ln\Gamma\left(1+\frac{\eta_s}{2}+i\frac{\omega}{2\pi T}\right)\right] \\ &= 2T\left(\frac{2\pi T}{D}\right)^{\eta_s}\sinh(\omega/2T)\frac{\omega/2T}{\sinh(\omega/2T)} \\ &\quad\times\exp\left[\eta_s\left(\mathrm{Re}\,\psi\left(1+i\frac{\omega}{2\pi T}\right)-\psi(2)\right)\right].\end{aligned} \tag{21}$$

Here Re denotes real part and ψ the digamma function. Dividing the expression by ω and taking the limit $\omega\to 0$ we obtain $(2\pi T/De)^{\eta_s}$, which is the renormalization factor of $1/\tau_{ss}$ due to the Luttinger properties of the electron gas. The exponents $\eta_\uparrow$ and $\eta_\downarrow$ are presented in Fig. 2 as a function of the polarization $2m$ for $n=N_e/N_a=0.5$ and several values of U.

The calculation of the critical exponents for the transconductivity is more difficult, since the transconductivity is a sum over products of an expectation value and a correlation

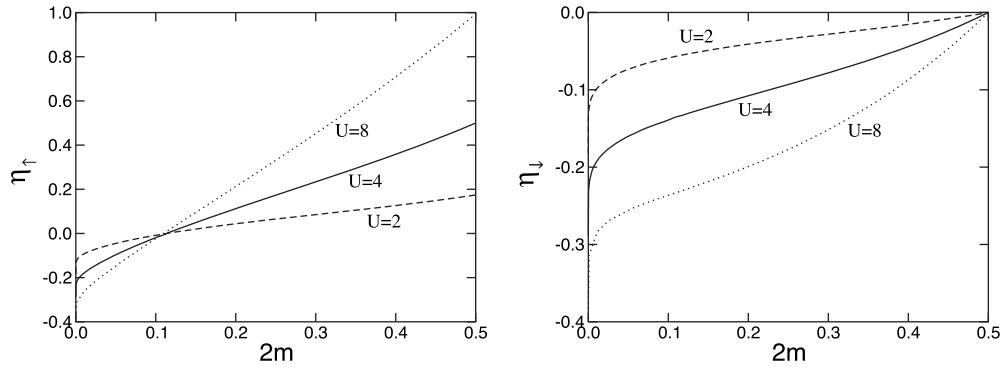

FIG. 2. Critical exponents η_s for the conductivity of up-spin carriers and down-spin carriers as a function of two times the magnetization for $n=N_e/N_a=0.5$ and $U=2$ (dashed curve), $U=4$ (solid curve) and $U=8$ (dotted curve), respectively.

function, Eq. (15). The expectation value $\langle a_{qs}^{\dagger} b_{ps} \rangle$ is determined through the Green's function $\langle\langle b_{ps}; a_{qs}^{\dagger} \rangle\rangle_{i\omega}$, which involves a product of two propagators, one for forward movers and one for backward movers. The renormalization of these two propagators needs to be calculated. The operators associated with these propagators are single fermion creation (annihilation) operators.

The quantum numbers for an up-spin propagator are $\Delta N_e=1$, $\Delta M=0$, $n_c^{\pm}=n_s^{\pm}=0$, and $D_c=-D_s=\pm 1/2$. This corresponds to a momentum transfer of $\Delta P=\pm p_{F\uparrow}$. The conformal dimensions are

$$2\Delta_c^{\pm}=\frac{1}{4}[z_{cc}-z_{sc}\pm z_{ss}/det]^2,$$

$$2\Delta_s^{\pm}=\frac{1}{4}[z_{cs}-z_{ss}\mp z_{sc}/det]^2. \tag{22}$$

Similarly, the quantum numbers for a down-spin propagator are $\Delta N_e=1$, $\Delta M=1$, $n_c^{\pm}=n_s^{\pm}=0$, $D_c=0$, and $D_s=\pm 1/2$. The momentum transfer in this case is $\Delta P=\pm p_{F\downarrow}$ and the conformal dimensions are

$$2\Delta_c^{\pm}=\frac{1}{4}[z_{sc}\pm (z_{ss}-z_{cs})/det]^2,$$

$$2\Delta_s^{\pm}=\frac{1}{4}[z_{ss}\pm (z_{cc}-z_{sc})/det]^2. \tag{23}$$

The propagator is now proportional to $[-i(\pi T/L)/\sinh(\pi Tt)]^{q_s^2}$, where $q_s^2=2(\Delta_c^+ +\Delta_c^- +\Delta_s^+ +\Delta_s^-)$ and t is the real time. Here we again assumed the long-time approximation, i.e., $|x|\ll vt$. Fourier transforming in analogy to Eq. (20) we obtain

$$-\frac{1}{\pi}\int dt\,\sin(\omega t)\left[\frac{-i\pi T/D}{\sinh(\pi Tt)}\right]^{q_s^2} = \frac{1}{2\pi^2 T}\frac{1}{\Gamma(q_s^2)}\left(\frac{2\pi T}{D}\right)^{q_s^2}\sinh(\omega/2T)\left|\Gamma\left(\frac{q_s^2}{2}+i\frac{\omega}{2\pi T}\right)\right|^2. \tag{24}$$

For the noninteracting system $q_s^2=1$, so that we write $q_s^2=1+\epsilon_s$ and expand the exponents in powers of ϵ_s

$$\sinh(\omega/2T)\frac{1}{2\pi^2 T}\frac{\pi}{\cosh(\omega/2T)}\frac{2\pi T}{D}\left(\frac{2\pi T}{D}\right)^{\epsilon_s} \times \exp\left[\epsilon_s\left(\mathrm{Re}\ \psi\left(\frac{1}{2}+i\frac{\omega}{2\pi T}\right)-\psi(1)\right)\right]. \tag{25}$$

This function corresponds to the imaginary part of the commutator correlation function, while we actually need the anticommutator function. The two functions are related via the fluctuation-dissipation theorem. Multiplying Eq. (25) by $\coth(\omega/2T)$ we finally obtain in the limit $\omega\to 0$

$$\frac{1}{D}\left(\frac{2\pi T}{D}\exp[\psi(1/2)-\psi(1)]\right)^{\epsilon_s}=\frac{1}{D}\left(\frac{\pi T}{2D}\right)^{\epsilon_s}. \tag{26}$$

The factor $(\pi T/2D)^{\epsilon_s}$ corresponds to the reduction of the discontinuity of the Fermi function at the Fermi level due to the Luttinger properties of the electron gas, i.e., for $T\to 0$ the discontinuity is suppressed with a nonuniversal power law (marginal Fermi liquid).

The correlation functions in Eq. (15) are the same ones we already calculated for the conductivities. Collecting all the factors, the transconductivity correlation function yields

$$\frac{1}{\tau_{\uparrow\downarrow}}=\frac{\lambda^2 U n_i m^*}{\pi^2 k_{F\downarrow}}\left(\frac{\pi T}{2D}\right)^{2\epsilon_{\downarrow}}\left(\frac{2\pi T}{eD}\right)^{\eta_{\uparrow}} + \frac{\lambda^2 U n_i m^*}{\pi^2 k_{F\uparrow}}\left(\frac{\pi T}{2D}\right)^{2\epsilon_{\uparrow}}\left(\frac{2\pi T}{eD}\right)^{\eta_{\downarrow}}, \tag{27}$$

so that the two critical exponents are, $\eta_{t1}=\eta_{\uparrow}+2\epsilon_{\downarrow}$ and $\eta_{t2}=\eta_{\downarrow}+2\epsilon_{\uparrow}$. These exponents are displayed in Fig. 3 for the same parameters as in Fig. 2.

V. DISCUSSION

(1) The dressed generalized charges as a function of the polarization are quite similar for the different values of U. They can be characterized by the limiting cases $2m\to 0$ and $2m\to 0.5$, which in the present case is the maximum polarization. For $2m\to 0$ the nonuniversal charge is $z_{cc}=2z_{sc}$, which with increasing U varies from $\sqrt{2}$ to 1. On the other

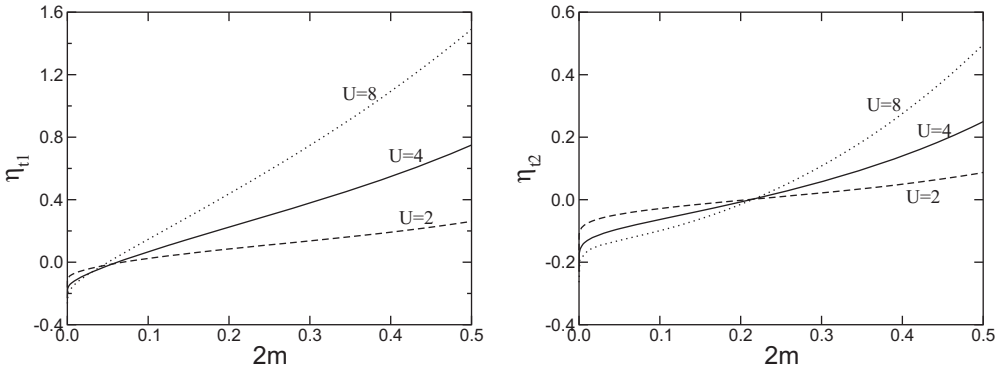

FIG. 3. Critical exponents for the transconductivity η_{t1} and η_{t2} as a function of two times the magnetization for $n=N_e/N_a=0.5$ and $U=2$ (dashed curve), $U=4$ (solid curve) and $U=8$ (dotted curve), respectively.

hand, for $2m\to 0.5$, z_{cs} is nonuniversal decreasing from 1.0 to 0.0 with increasing U.

(2) As expected, the relaxation rate for the up-spin and down-spin conductivity are equal to each other in the absence of spin-polarization. For $2m=0$ the exponents are negative, i.e., $1/\tau_{ss}$ diverges with a power law as $T\to 0$. The exponent varies from 0 (for $U=0$) to -0.5 (for $U=\infty$). As a function of $2m$ the exponents for the minority spins increase but remain negative for all polarizations and U. For the majority spin carriers the exponents increase monotonically with $2m$ and change sign at about $2m=0.11$. The variation of $\eta_\uparrow$ with $2m$ is larger for large U. Hence, the majority spin relaxation rate decreases as $T\to 0$ for large spin polarization, while it increases for weak spin polarization.

(3) As in Ref. 14 there are two exponents for the transconductivity, which increase monotonically with $2m$. The rate of increase depends on U, being larger for the strongly interacting system. For $2m\to 0$ the exponents are negative, so that $1/\tau_{\uparrow\downarrow}$ increases as $T\to 0$, while for strong spin polarization the exponents are positive and $1/\tau_{\uparrow\downarrow}$ decreases as $T\to 0$.

(4) The exponent $\eta_\uparrow$ in Fig. 2 appears to change sign at a value of the magnetization that is independent of U. A detailed analysis of the exponent in the neighborhood of that point reveals that (i) the three curves shown do not intersect each other exactly at the same point, and (ii) the vanishing of $\eta_\uparrow$ does not occur at exactly the same magnetization for the three values of U. The same is true for η_{t2} in Fig. 3.

(5) In Ref. 14 we calculated the exponents within the bosonization approximation as a function of a forward-scattering interaction V and a backward-scattering amplitude $U_\parallel$. All exponents were found to depend strongly on the forward-scattering amplitude and if V is sufficiently strong all exponents are positive. The spin-polarization was kept fixed, since it is difficult to vary $2m$ within the bosonization approach. The bosonization technique yields asymptotically exact exponents within the weak-coupling limit.

(6) We reproduced the results by Luther and Peschel[26] for the conductivity of spinless fermions with impurities and forward scattering. This case, obviously, refers to no spin polarization.

(7) The Coulomb interaction in the Hubbard model is a local interaction, which confines the forward and backward-scattering amplitudes to be equal, $V=U_\parallel$. In order to compare the present Bethe *Ansatz* and conformal field theory calculation with the bosonization results, it is necessary to consider the $V=U_\parallel$ weak-coupling limit. For zero spin polarization the bosonization calculation also yields that $\eta_\uparrow=\eta_\downarrow$. For weak coupling and spin polarization both exponents are very small and positive. For the Bethe *Ansatz* solution the exponent has the same order of magnitude but is negative. The small difference between the two calculations can be attributed to $U_\perp$ (spin-flip scattering across the Fermi surface), which does not contribute to the bosonization exponents, but it does in the case of the Bethe *Ansatz* solution. $U_\perp$ provides another channel for spin-transfer between the up- and down- spin currents. Hence, the exponents are expected to be reduced and in this case even change sign.

(8) The present Bethe *Ansatz* calculation is different from the one carried out for the Coulomb drag between two nearby parallel wires.[27] The spin Coulomb drag is based on scattering off impurities and the Coulomb interaction, while in the Coulomb drag the momentum transfer is between wires but no impurities are required.

(9) There are two relaxation rate exponents for the transconductivity in both calculations (bosonization and Bethe *Ansatz*). As the Coulomb interactions tend to zero, both exponents vanish in both calculations. A comparison between the exponents for weak coupling in the presence of spin polarization yields that they are small in both cases and a discrepancy may be attributed to the spin-flip scattering across the Fermi surface ($U_\perp$), which does not contribute to the exponents in the bosonization case.

(10) We now invert the conductivity matrix to obtain the dc resistivities,[21]

$$\rho_{ss'}=-\frac{(m^*)^2}{e^2 n_s n_{s'}}\lim_{\omega\to 0}\omega\langle\langle j_s;j_{s'}\rangle\rangle''_\omega. \tag{28}$$

This way the exponent of $\rho_{\uparrow\uparrow}$ is $\eta_\uparrow$, the one of $\rho_{\downarrow\downarrow}$ is $\eta_\downarrow$ and the exponents of the transresistivity are η_{t1} and η_{t2}. If the exponent η_s is positive the resistivity of the corresponding channel decreases as $T\to 0$ and the channel behaves metallic-like, while if the exponent is negative the resistivity of the channel increases as $T\to 0$ and the channel behaves like an insulator.

(11) The temperature dependence of the majority and minority spin resistivity and the transresistivity are shown in Fig. 4 for three values of the magnetization, normalized to the value for $T=D$. For strong spin polarization the majority

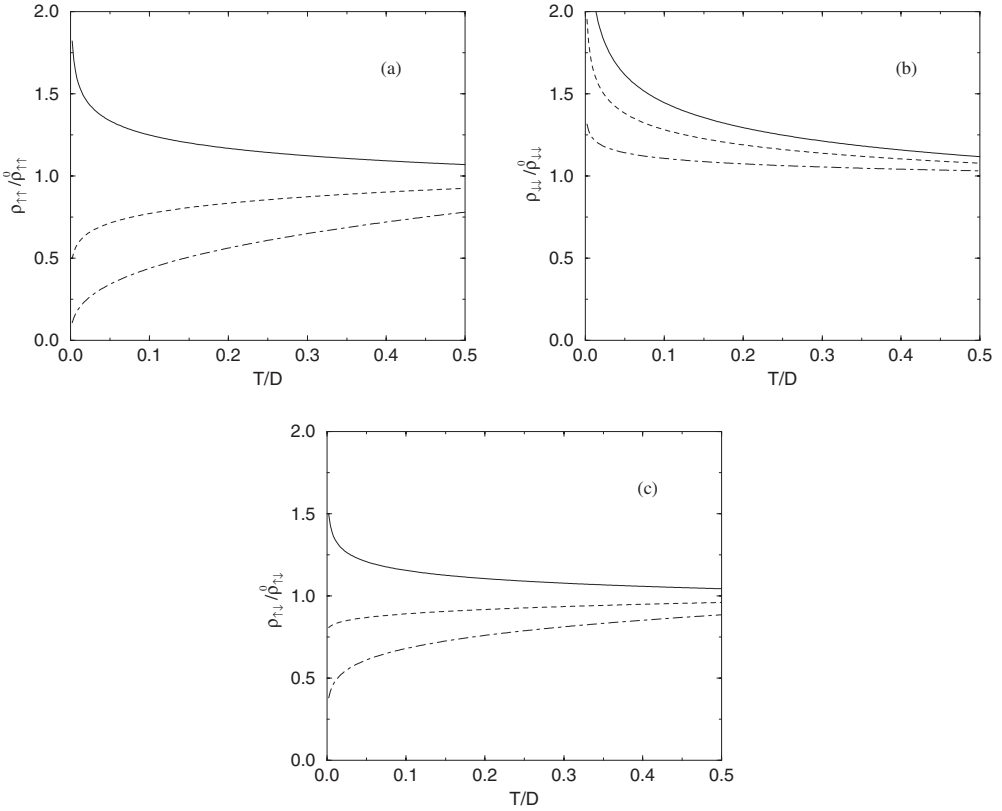

FIG. 4. Temperature dependence of the (a) majority spin resistivity, (b) minority spin resistivity, and (c) transresistivity normalized to their values for $T=D$ for $U=4$ and $n=0.5$. The three curves correspond to different magnetizations: For the solid line $2m=0.05$, for the dashed curve $2m=0.20$ and for the dash-dotted curve $2m=0.40$.

spin carriers display a metalliclike behavior, while for small spin polarization the temperature dependence suggests an insulator. The minority spins always tend to have a small conduction (insulatorlike). The transresistivity has positive exponents for large spin-polarization; this fact is favorable for a sustainable spin current. The transresistivity has two terms,

$$\frac{\rho_{\uparrow\downarrow}}{\rho^0_{\uparrow\downarrow}} = \frac{A_\uparrow (T/D)^{\eta_{t1}} + A_\downarrow (T/D)^{\eta_{t2}}}{A_\uparrow + A_\downarrow}, \tag{29}$$

where we parametrized $A_s = 1/(0.5+2sm)$ with $s=\pm 1$. It is assumed that the magnetization does not change with temperature.

(12) The power laws arising from the Luttinger liquid properties have the underlying assumption of the long-time approximation. However, the electron states are not infinitely lived due to the scattering off the impurities. Hence, the power laws are no longer valid below a characteristic temperature that depends on the scattering strength, and the impurity and carrier concentration. Below this characteristic temperature localization is expected to take over. In other words, the conductivities or transconductivity cannot become arbitrarily large if the corresponding exponent is positive.

(13) The calculation of the transconductivity involves the factorization shown in Eq. (16). This factorization is necessary to employ conformal field theory to evaluate the correlation function. Due to this factorization the results obtained for the transconductivity are only approximate. The same factorization was used for the calculation within the bosonization approach, where it is asymptotically exact within the long-time approximation.

VI. CONCLUSIONS

We studied the spin Coulomb drag between the majority and minority spin components in a spin polarized quantum wire. As shown in Ref. 14 the momentum relaxation, i.e., the increase of the momentum of the minority carriers at the expense of the majority electrons without change in the magnetization, is only effective with back-scattering off nonmagnetic impurities. We have neglected the interference in the scattering between impurities and hence excluded all possibility for a localization of states due to disorder.

The temperature dependence of the conductivity and transconductivity in the form of power laws is introduced by the Luttinger liquid properties of the 1D Hubbard model. Within the bosonization approach for a nonlocal interaction the dominant scattering amplitude is forward scattering V. For the Hubbard model the forward and backward-scattering amplitudes are equal. This leads to a situation unfavorable for conduction at low T for small spin polarization. For large spin polarization the majority spin carriers are favorable to conduction, while the minority spin electrons have a large resistivity. The spin Coulomb drag is proportional to the transresistivity,[1] which at low T is large for small spin-polarization while small for large polarization.

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